

4.7) Να απαντήσετε σε

$$\text{i)} \lim_{x \rightarrow +\infty} a^x = \begin{cases} +\infty, & a > 1 \\ 1, & a = 1 \\ 0, & 0 < a < 1 \end{cases}$$

$$\text{ii)} \lim_{x \rightarrow -\infty} a^x = \begin{cases} 0, & a > 1 \\ 1, & a = 1 \\ +\infty, & 0 < a < 1 \end{cases}$$

ΑΝΣΩ

i) $a > 1$, τότε $a = 1 + \theta, \theta > 0 \Rightarrow a^v = (1 + \theta)^v \geq 1 + \theta v > \theta v$ (Σελ 48)

Άρα, $\lim a^v = +\infty \Leftrightarrow (\forall \epsilon > 0)(\exists v_0 \in \mathbb{N})(\forall v \in \mathbb{N}) v > v_0 \Rightarrow a^v > \frac{1}{\epsilon}$
 $(v_0 + 1 > v_0) \Rightarrow (a^{v_0+1} > \frac{1}{\epsilon})$

Έστω, $x_v \rightarrow +\infty$ και βρισκούμε $\lim a^{x_v}$

$$\text{αφού } x_v \rightarrow +\infty \Leftrightarrow (\forall \epsilon^* > 0)(\exists v_1 \in \mathbb{N})(\forall v \in \mathbb{N}) v > v_1 \Rightarrow x_v > \frac{1}{\epsilon^*} \equiv v_0 + 1$$
$$\Rightarrow a^{x_v} > a^{v_0+1} > \frac{1}{\epsilon^*} \Rightarrow \lim a^{x_v} = +\infty \Rightarrow \lim_{x \rightarrow +\infty} a^x = +\infty$$

• $a = 1$, προπάνες

$$\bullet 0 < a < 1 \quad (\text{Ζετείται}) \Rightarrow |a| < 1 \Rightarrow \frac{1}{|a|} > 1 \Rightarrow \frac{1}{|a|^v} = 1 + \theta, \theta > 0 \Rightarrow$$
$$\Rightarrow \frac{1}{|a|^v} = (1 + \theta)^v \geq 1 + v\theta > v\theta \quad |a| \quad |a|$$

Άρα, $\lim a^v = 0 \Leftrightarrow (\forall \epsilon > 0)(\exists v_0 \in \mathbb{N})(\forall v \in \mathbb{N}) v > v_0 \Rightarrow a^v < \epsilon$
 $(v_0 + 1 > v_0) \Rightarrow (a^{v_0+1} < \epsilon)$

Έστω, $x_v \rightarrow +\infty$ και βρισκούμε $\lim a^{x_v}$

$$\text{αφού } x_v \rightarrow +\infty \Leftrightarrow (\forall \epsilon^* > 0)(\exists v_1 \in \mathbb{N}) : v > v_1 \Rightarrow x_v > \frac{1}{\epsilon^*} \equiv v_0 + 1$$
$$\Rightarrow a^{x_v} < a^{v_0+1} < \epsilon \Rightarrow \lim a^{x_v} = 0 \Rightarrow \lim_{x \rightarrow +\infty} a^x = 0$$

ii) $\circ a > 1$, τότε $a = 1 + \theta$, $\theta > 0 \Rightarrow a^v = (1 + \theta)^v \geq 1 + v\theta > v\theta$.

Αριθμ. $\lim a^v = +\infty \Leftrightarrow (\forall \varepsilon > 0)(\exists N \in \mathbb{N})(\forall v > N) : v > N \Rightarrow a^v > \frac{1}{\varepsilon}$

Εσω $x_v \rightarrow -\infty$ και θα βρούμε $\lim a^{x_v}$ ($v_0 + 1 > v_0$) $\Rightarrow (a^{v_0+1} > \frac{1}{\varepsilon})$

Αφού $x_v \rightarrow -\infty \Leftrightarrow (\forall \varepsilon^* > 0)(\exists N \in \mathbb{N})(\forall v < N) : v > N \Rightarrow x_v < -\frac{1}{\varepsilon^*} = -(v_0 + 1)$

$$\Rightarrow a^{x_v} < a^{-(v_0+1)} < \frac{1}{\varepsilon^*} \Rightarrow \text{με } \frac{1}{a^{v_0+1}} > \varepsilon$$

Αριθμ. $a^{x_v} < \varepsilon \Rightarrow \lim a^{x_v} = 0 \quad a^{v_0+1}$

$$\Rightarrow \lim_{x \rightarrow -\infty} a^x = 0$$

$\circ a = 1$, προφανές

$\circ 0 < a < 1 \Rightarrow |a| < 1 \Rightarrow \frac{1}{|a|} > 1 \Rightarrow \frac{1}{|a|^v} = 1 + \theta, \theta > 0 \Rightarrow \frac{1}{|a|^v} = (1 + \theta)^v \geq 1 + v\theta > v\theta$

Αριθμ. $\lim a^v = 0 \Leftrightarrow (\forall \varepsilon > 0)(\exists N \in \mathbb{N})(\forall v > N) : v > N \Rightarrow a^v < \varepsilon$

$$(v_0 + 1 > N_0) \Rightarrow (a^{v_0+1} < \varepsilon)$$

Οι βρου το $\lim a^{x_v}$ για $x_v \rightarrow -\infty$

Αφού $x_v \rightarrow -\infty \Leftrightarrow (\forall \varepsilon^* > 0)(\exists N \in \mathbb{N})(\forall v < N) : v > N \Rightarrow x_v < -\frac{1}{\varepsilon^*} = -(v_0 + 1)$

$\Rightarrow x_v > -(v_0 + 1) \Rightarrow a^{x_v} > a^{-(v_0+1)} > \frac{1}{\varepsilon^*} \Rightarrow a^{x_v} > \frac{1}{\varepsilon} \Rightarrow$

$$\Rightarrow \lim a^{x_v} = +\infty \Rightarrow \lim_{x \rightarrow -\infty} a^x = +\infty \quad \text{όπου } a^{-(v_0+1)} > \frac{1}{\varepsilon}$$